Note on Fundamentals of Physics

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SDU

Contents

Chapter 1

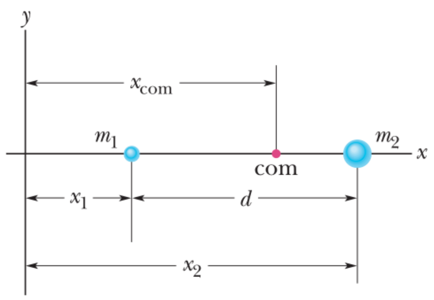
Center of Mass and Linear Momentum (No problems)

1.1 The Center of Mass

▶ The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

1.1.1 System of particles

Look at this picture:



Hence we have

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

Now we have three dimensions case:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

And vector form:

$$\mathbf{r} = \frac{1}{M}\sum_{cyc}\sum_{i=1}^{n} m_i x_i \mathbf{i} = \frac{1}{M}\sum_{i=1}^{n} m_i \mathbf{r_i}$$

where $\mathbf{r}_i = x_i\mathbf{i} + y_i\mathbf{j} + z_i\mathbf{k}$.

1.1.2 General solid bodies case

We have:

$$x_{com} = \frac{1}{M}\int_{\mathfrak{M}} xdm, y_{com} = \frac{1}{M}\int_{\mathfrak{M}} ydm, z_{com} = \frac{1}{M}\int_{\mathfrak{M}} zdm.$$

And vector form:

$$\mathbf{r}_{com} = \frac{1}{M}\int_{\mathfrak{M}} \mathbf{r}dm.$$

If ρ is a constant, then we have:

$$x_{com} = \frac{1}{V}\int_{\Omega} xdV, y_{com} = \frac{1}{V}\int_{\Omega} ydV, z_{com} = \frac{1}{V}\int_{\Omega} zdV.$$

1.2 Newton's Second Law for a System of Particles

Since $M\mathbf{r}_{com} = \sum_{k=1}^{n} m_k\mathbf{r}_k$ and $\mathbf{v} = \frac{d\mathbf{r}}{dt}$, we have:

$$M\mathbf{v}_{com} = \sum_{k=1}^{n} m_k\mathbf{v}_k.$$

And since $\mathbf{a} = \frac{d\mathbf{v}}{dt}$, we have internal forces $= 0$ and external forces:

$$\mathbf{F}_{net} := M\mathbf{a}_{com} = \sum_{k=1}^{n} m_k\mathbf{a}_k = \sum_{k=1}^{n} \mathbf{F}_k.$$

And

$$\mathbf{F}_{net,x} = M\mathbf{a}_{com,x}, \mathbf{F}_{net,y} = M\mathbf{a}_{com,y}, \mathbf{F}_{net,z} = M\mathbf{a}_{com,z}.$$

1.3 Linear Momentum

1.3.1 Linear Momentum

We have $\mathbf{p} = m\mathbf{v}$ and $\mathbf{F}_{net} = \frac{d\mathbf{p}}{dt}$.

1.3.2 The Linear Momentum of a System of Particles

We have $\mathbf{p} = M\mathbf{v}_{com}$ and $\mathbf{F}_{net} = \frac{d\mathbf{p}}{dt} = M\mathbf{a}_{com}$.

1.4 Collision and Impulse

We have $\mathbf{F} = \frac{d\mathbf{p}}{dt}$, hence $d\mathbf{p} = \mathbf{F}dt$, then

$$\Delta\mathbf{p} = \int_{t_0}^{t_1} \mathbf{F}dt.$$

We define $F_{avg} = \frac{\Delta\mathbf{p}}{\Delta t}$ and impulse $\mathbf{J} = \int_{t_0}^{t_1} \mathbf{F}dt = F_{avg}\Delta t$.

1.5 Conservation of Linear Momentum

In a closed, isolated system, we have $\mathbf{F}_{net} = \mathbf{0}$, hence $\mathbf{p} = C\mathbf{e}$, where $|\mathbf{e}| = 1$.

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

1.6 Momentum and Kinetic Energy in Collisions

►Here, momentum is constant in all collisions:

1.elastic collision: kinetic energy of the system is conserved.

2.inelastic collision: kinetic energy of the system is not conserved.

3.completely inelastic collision: greatest loss of kinetic energy, i.e., particles stick together.

COM: center of mass of the system cannot be changed by a collision because, with the system isolated, there is no net external force to change it.

1.6.1 completely inelastic collision in one dimension

We have

$$\sum_{k=1}^{n} m_k \mathbf{v}_k = M\mathbf{v}$$

hence

$$\mathbf{v} = \frac{1}{M}\sum_{k=1}^{n} m_k \mathbf{v}_k.$$

1.7 Elastic Collisions in One Dimension

Two balls case:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}, \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

hence we have:

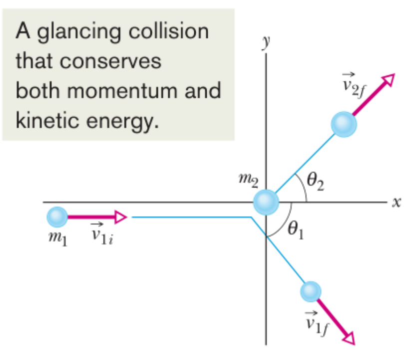
$$v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2v_{2i}}{m_1 + m_2}, v_{2f} = \frac{2m_1v_{1i} + (m_2 - m_1)v_{2i}}{m_1 + m_2}.$$

1.8 Collisions in Two Dimensions

Elastic collision:

1.total kinetic energy is also conserved

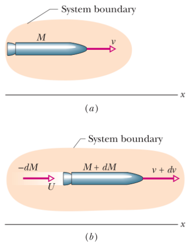
2.linear momentum must still be conserved



Then we have $m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$ and $m_1 v_{1f} \sin\theta_1 = m_2 v_{2f} \sin\theta_2$.

1.9 Systems with Varying Mass: A Rocket

Look at this picture: We have $Mv = -UdM + (M + dM)(v + dv)$.



Use relative speed, we have $U = v + dv - v_{rel}$, hence $-v_{rel}dM = Mdv$, then $-v_{rel}\dfrac{dM}{dt} = M\dfrac{dv}{dt}$. So we have $Rv_{rel} = Ma$ which we call first rocket equation.

Note the left side of the first rocket equation has the dimensions of force and depends only on design characteristics of the rocket engine-namely, the rate R at which it consumes fuel mass and the speed v_{rel} with which that mass is ejected relative to the rocket. We call this term Rv_{rel} the thrust of the rocket engine and represent it with T. Newton's second law emerges if we write the first rocket equation as $T = Ma$, in which a is the acceleration of the rocket at the time that its mass is M.

Use conserve momentum or relative speed, we have $dv = -v_{rel}\dfrac{dM}{M}$, then $v_f - v_i = v_{rel} \ln \dfrac{M_i}{M_f}$.

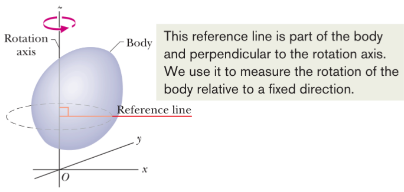
Chapter 2

Rotation

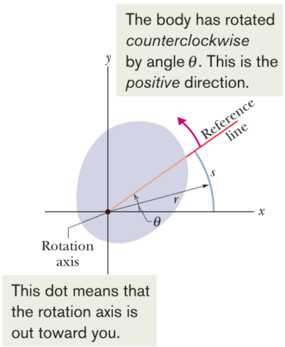
2.1 Rotational Variables

►A rigid body is a body that can rotate with all its parts locked together and without any change in its shape.

►A fixed axis means that the rotation occurs about an axis that does not move.



This figure shows a rigid body of arbitrary shape in rotation about a fixed axis, called the axis of rotation or the rotation axis.



The angular position:

$$\theta = \frac{s}{r}.$$

The angular displacement:

$$\Delta\theta = \theta_2 - \theta_1.$$

The angular velocity:

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}, \omega = \frac{d\theta}{dt}.$$

The angular acceleration:

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}, \alpha = \frac{d\omega}{dt}.$$

It's necessary to know that angular velocity is a vector which follows right-hand rule.

2.2 Rotation with Constant Angular Acceleration

When α is a constant, we know that

$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega_f = \omega_i + \alpha t \Rightarrow \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

Then

$$\omega_f^2 - \omega_i^2 + 2\alpha(\theta_f - \theta_i).$$

Beyond that, we have

$$\theta_f - \theta_i = \frac{1}{2}(\omega_i + \omega_f)t = \omega_f t - \frac{1}{2}\alpha t^2.$$

2.3 Relating the Linear and Angular Variables

►The position:

$$s = \theta r.$$

►The speed:

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v = \omega r.$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}.$$

►The acceleration:

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = \alpha r = \frac{v^2}{r} = \omega^2 r.$$

2.4 Kinetic Energy of Rotation

►Rotational inertia: (i) Particles:

$$I = \sum_i m_i r_i^2.$$

(ii) Commutative rigid body:

$$I = \iiint_V r^2 dm = \iiint_V r^2 \rho dV.$$

And we have

$$K = \frac{1}{2}I\omega^2.$$

2.5 Calculating the Rotational Inertia

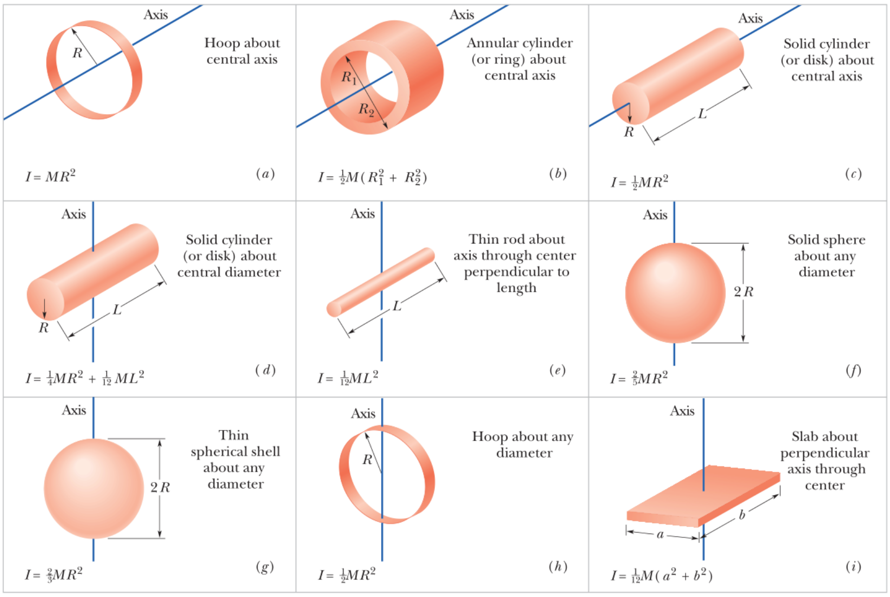
We have

$$I = \iiint_V r^2 dm = \iiint_V r^2 \rho dV.$$

And some useful cases:

★ **Theorem 2.5.1**

(Parallel-Axis Theorem) Suppose we want to find the rotational inertia I of a body of mass M about a given axis. In principle, we can always find I with the integration of $\iiint_V r^2 dm = \iiint_V r^2 \rho dV$. However,there is a neat shortcut if we happen to already know the rotational inertia I_{com} of the body about a parallel axis that extends through the body's center of mass. Let h be the perpendicular

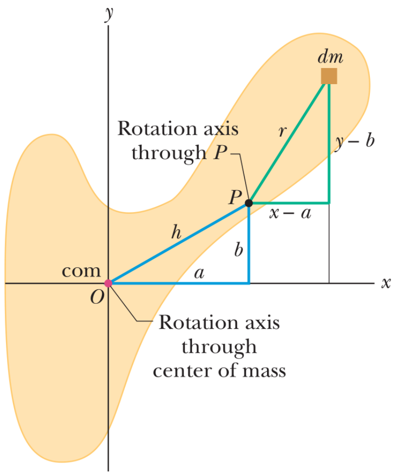


distance between the given axis and the axis through the center of mass (remember these two axes must be parallel). Then the rotational inertia I about the given axis is

$$I = I_{com} + Mh^2$$

Think of the distance h as being the distance we have shifted the rotation axis from being through the com.

Proof. Let O be the center of mass of the arbitrarily shaped body shown in cross section in Figure. Place the origin of the coordinates at O. Consider an axis through O perpendicular to the plane of the figure, and another axis through point P parallel to the first axis. Let the x and y coordinates of P be a and b.

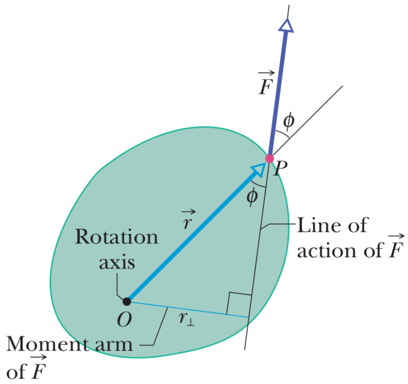


Let dm be a mass element with the general coordinates x and y. The rotational inertia of the body about the axis through P is then,

$$I = \int_V r^2 dm = \int_V ((x-a)^2 + (y-b)^2) dm$$
$$= \int_V (x^2+y^2) dV - 2a\int_V x dm - 2b \int_V y dm + \int_V (a^2+b^2) dm$$
$$= \int_V r^2 dV - \frac{2ax_{com}}{M} - \frac{2by_{com}}{M} + Mh^2 = I_{com} + Mh^2.$$

□

2.6 Torque



We define the torque by $\tau = rF\sin\phi$ and $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. And more information is in the picture.

Clocks Are Negative. If a torque would cause counterclockwise rotation, it is positive. If it would cause clockwise rotation,it is negative.

Net Torque. We have that

$$\boldsymbol{\tau}_{net} = \sum_{k=1}^{n} \boldsymbol{\tau}_k.$$

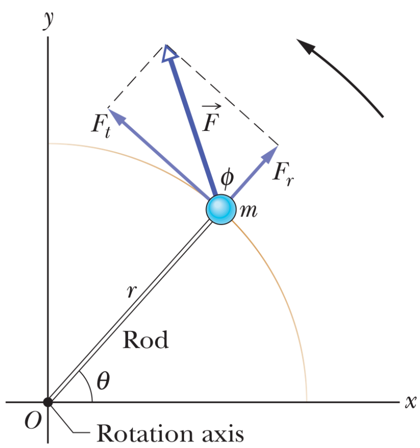
2.7 Newton's Second Law for Rotation

★ **Theorem 2.7.1**

(Newton's Second Law for Rotation) A torque can cause rotation of a rigid body, as when you use a torque to rotate a door. Then we have

$$\tau_{net} = I\alpha.$$

Proof. Use the picture



We have

$$\tau = F_t r = ma_t r = mr^2\alpha = I\alpha.$$

If more than one force is applied to the particle, it becomes

$$\tau_{net} = I\alpha.$$

□

2.8 Work and Rotational Kinetic Energy

Actually, we can calculate the work:

$$W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2,$$

In the other hand, we can calculate the work:

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta.$$

When τ is constant, we have

$$W = \tau(\theta_f - \theta_i).$$

The rate at which the work is done is the power,which we can find that

$$P = \frac{dW}{dt} = \tau\omega.$$

Chapter 3

Rolling, Torque, and Angular Momentum

3.1 Rolling as Translation and Rotation Combined

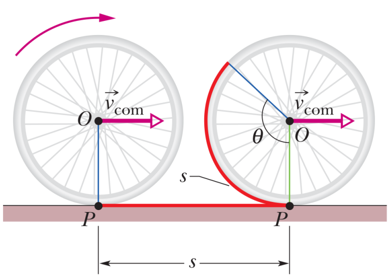
★ Theorem 3.1.1

For a wheel of radius R rolling smoothly,

$$v_{com} = \omega R$$

where v_{com} is the linear speed of the wheel's center of mass and ω is the angular speed of the wheel about its center.

Proof. Look at the picture

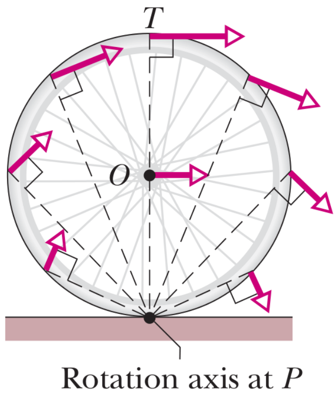


Then we have $s = \theta R$ and $v_{com} = \dfrac{ds}{dt} = R\dfrac{d\theta}{dt} = \omega R$. □

► Actually, we have

$$\mathbf{v}_{rel} + \mathbf{v}_{vom} = \mathbf{v}.$$

And we have this interesting thing:



3.2 Forces and Kinetic Energy of Rolling

3.2.1 The Kinetic Energy of Rolling

★ Theorem 3.2.1

A smoothly rolling wheel has kinetic energy

$$K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{vom}^2$$

where I_{com} is the rotational inertia of the wheel about its center of mass and M is the mass of the wheel.

Proof. We know that

$$K = \frac{1}{2} I_P \omega^2$$

in which ω is the angular speed of the wheel and I_P is the rotational inertia of the wheel about the axis through P. From the parallel-axis theorem, we have

$$K = \frac{1}{2} I_P \omega^2 = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M R^2 \omega^2 = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{vom}^2.$$

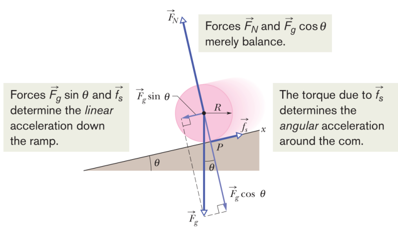
□

3.2.2 The Forces of Rolling

For smooth rolling we have

$$a_{com} = \alpha R.$$

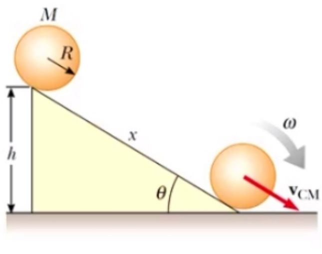
▶ Here we have an example:



So we have $f_s - Mg\sin\theta = Ma_{com,x}$ and $Rf_s = I_{com}\alpha$, then

$$a_{com,x} = -\frac{g\sin\theta}{1 + I_{com}/(MR^2)}.$$

▶ For the solid sphere shown in the picture, calculate the linear speed of the center of mass at the bottom of the incline and the magnitude of the linear acceleration of the center of mass.

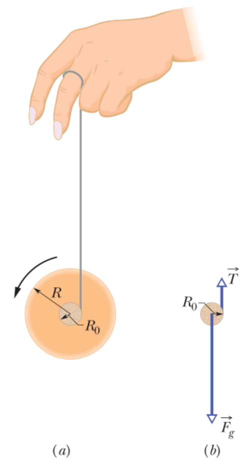


And we have

$$K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2 = Mgh \Rightarrow v_{CM} = \left(\frac{2gh}{1 + I_{CM}/(MR^2)}\right)^{1/2} = \sqrt{\frac{10}{7}gh} \Rightarrow a_{CM} = \frac{5}{7}g\sin\theta.$$

3.3 The Yo-Yo

Like this:



We will just let $\theta = 90°$ in the last section. So we have

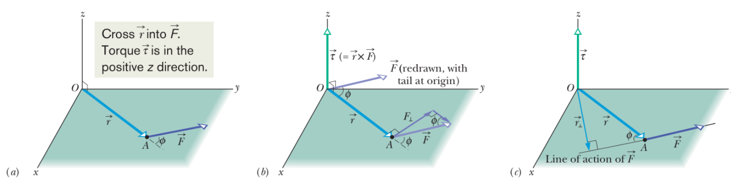
$$a_{com,x} = -\frac{g}{1 + I_{com}/(MR^2)}.$$

3.4 Torque Revisited

We have $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ where $\mathbf{F}$ is a force applied to a particle and $\mathbf{r}$ is a position vector locating the particle relative to the fixed point.

And actually, we have $|\boldsymbol{\tau}| = |\mathbf{r}||\mathbf{F}|\sin\langle\mathbf{r}, \mathbf{F}\rangle = r_\perp F = rF_\perp$ where $F_\perp$ is the component of $\mathbf{F}$ perpendicular to $\mathbf{r}$, and $r_\perp$ is the moment arm of $\mathbf{F}$.

And the direction of is given by the right-hand rule for cross products. And:



3.5 Angular Momentum

The angular momentum $\boldsymbol{\ell}$ of a particle with linear momentum $\mathbf{p}$, mass m, and linear velocity $\mathbf{v}$ is a vector quantity defined relative to a fixed point (usually an origin) as $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$ where is the position vector of the particle with respect to O.

3.6 Newton's Second Law in Angular Form

★ Theorem 3.6.1

(Newton's Second Law in Angular Form) In single particle, we have

$$\boldsymbol{\tau}_{net} = \frac{d\boldsymbol{\ell}}{dt}.$$

Proof. Since $\mathbf{v} \times \mathbf{v} = \mathbf{0}$, we have

$$\frac{d\boldsymbol{\ell}}{dt} = m(\mathbf{r} \times \mathbf{a}) = \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times \mathbf{F}_{net} = \boldsymbol{\tau}_{net}.$$

□

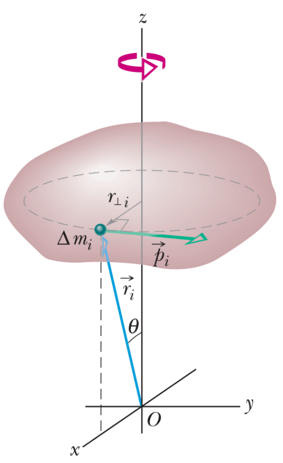
3.7 Angular Momentum of a Rigid Body

3.7.1 The Angular Momentum of a System of Particles

In a system of particles, we have $\mathbf{L} = \sum_i \boldsymbol{\ell}_i$ and $\boldsymbol{\tau}_{net} = \dfrac{d\mathbf{L}}{dt}$.

3.7.2 The Angular Momentum of a Rigid Body Rotating About a Fixed Axis

Look at this picture:



We are interested in the component of $\boldsymbol{\ell}_i$ that is parallel to the rotation axis, here the z axis.That z component is

$$\ell_{iz} = \ell \sin\theta = r_{\perp i} \Delta m_i v_i.$$

Thus

$$L_z = \sum_i \ell_{iz} = \sum_i r_{\perp i} \Delta m_i v_i = \sum_i r_{\perp i}^2 \Delta m_i \omega_i = I\omega.$$

So we have

$$L = I\omega.$$

3.8 Conservation of Angular Momentum

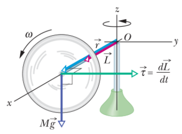
The angular momentum $\mathbf{L}$ of a system remains constant if the net external torque acting on the system is zero:

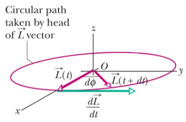
$$\mathbf{L} = Constant \Rightarrow \mathbf{L}_i = \mathbf{L}_f.$$

Also, we can have this:

$$I_i\omega_i = I_f\omega_f.$$

3.9 Precession of a Gyroscope





We have $dL = \tau dt = Mgrdt$ and $d\phi = \dfrac{dL}{L} = \dfrac{Mgrdt}{I\omega}$, hence $\Omega = \dfrac{d\phi}{dt} = \dfrac{Mgr}{I\omega}$.

Chapter 4

Equilibrium and Elasticity

4.1 Equilibrium

★ Theorem 4.1.1

A rigid body at rest is said to be in static equilibrium. For such a body, the vector sum of the external forces acting on it is zero:

$$\boldsymbol{F}_{net} = \mathbf{0}.$$

Notation 4.1.1. If all the forces lie in the x, y plane, this vector equation is equivalent to two component equations:

$$F_{net,x} = 0, F_{net,y} = 0.$$

★ Theorem 4.1.2

Static equilibrium also implies that the vector sum of the external torques acting on the body about any point is zero, or

$$\boldsymbol{\tau}_{net} = \mathbf{0}.$$

Notation 4.1.2. If the forces lie in the x, y plane, all torque vectors are parallel to the z axis, and the balance-of-torques equation is equivalent to the single component equation:

$$\tau_{net,z} = 0.$$

► The gravitational force acts individually on each element of a body. The net effect of all individual actions may be found by imagining an equivalent total gravitational force acting at the center of gravity. If the gravitational acceleration is the same for all the elements of the body, the center of gravity is at the center of mass.

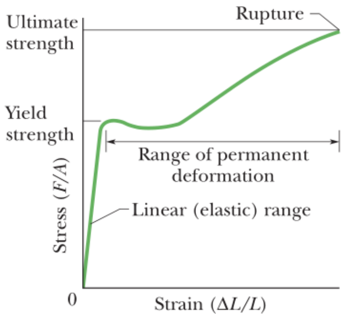
4.2 Elasticity

Three elastic moduli are used to describe the elastic behavior (deformations) of objects as they respond to forces that act on them. The strain (fractional change in length) is linearly related to the applied stress (force per unit area) by the proper modulus, according to the general stress-strain relation stress = modulus × strain.

1. Young's modulus: When an object is under tension or compression, the stress-strain relation is written as

$$E = \frac{F/A}{\Delta L/L_i},$$

where $\Delta L/L_i$ is the tensile or compressive strain of the object, F is the magnitude of the applied force $\boldsymbol{F}$ causing the strain, A is the cross-sectional area over which $\boldsymbol{F}$ is applied (perpendicular to A), and E is the Young' s modulus for the object. The stress is F/A.



2. Shear modulus: When an object is under a shearing stress, the stress-strain relation is written as

$$S = \frac{F/A}{\Delta x/L},$$

where $\Delta x/L$ is the shearing strain of the object, Δx is the displacement of one end of the object in the direction of the applied force $\boldsymbol{F}$, and G is the shear modulus of the object. The stress is F/A.

3. Bulk modulus: When an object undergoes hydraulic compression due to a stress exerted by a surrounding fluid, the stress-strain relation is written as

$$B = \frac{p}{\Delta V/V},$$

where p is the pressure (hydraulic stress) on the object due to the fluid, $\Delta V/V$ (the strain) is the absolute value of the fractional change in the object's volume due to that pressure, and B is the bulk modulus of the object.

Chapter 5

Gravitation

5.1 Newton's Law of Gravitation

★ **Theorem 5.1.1**

(Newton's law of gravitation) Any particle in the universe attracts any other particle with a gravitational force whose magnitude is

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the particles, r is their separation, and $G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$ is the gravitational constant.

Notation 5.1.1. 1. Actually, we can write the law in vector form: $\boldsymbol{F} = -G\frac{m_1 m_2}{r^2} \cdot \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$.

2. A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

5.2 Gravitation and the Principle of Superposition

5.2.1 Particles

Gravitational forces obey the principle of superposition; that is, if n particles interact, the net force $\boldsymbol{F}_{1,net}$ on a particle labeled particle 1 is the sum of the forces on it from all the other particles taken one at a time:

$$\boldsymbol{F}_{1,net} = \sum_{k=2}^{n} \boldsymbol{F}_{1,i}.$$

5.2.2 Body

The gravitational force $\boldsymbol{F}_1$ on a particle from an extended body is found by first dividing the body into units of differential mass dm, each of which produces a differential force $d\boldsymbol{F}$ on the particle, and then integrating over all those units to find the sum of those forces:

$$\boldsymbol{F}_1 = \int d\boldsymbol{F}.$$

5.3 Gravitation Near Earth's Surface

Let us assume that Earth is a uniform sphere of mass M. The magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth a distance r from Earth's center, is then given by $F = G\frac{Mm}{r^2}$. Then $a_g = \frac{GM}{r^2}$.

If something on the surface with m, then we have $ma_g = mg + m\omega^2 r$ and $a_g = g + \omega^2 r$.

5.4 Gravitation Inside Earth

▶ A uniform shell of matter exerts no net gravitational force on a particle located inside it.

▶ The gravitational force $\boldsymbol{F}$ on a particle inside a uniform solid sphere, at a distance r from the center, is due only to mass M ins in an "inside sphere" with that radius r:

$$M_{ins} = \frac{4}{3}\pi r^3 \rho = \frac{M}{R^3} r^3$$

where ρ is the solid sphere's density, R is its radius, and M is its mass. We can assign this inside mass to be that of a particle at the center of the solid sphere and then apply Newton's law of gravitation for particles. We find that the magnitude of the force acting on mass m is

$$F = \frac{GMm}{R^3} r.$$

5.5 Gravitational Potential Energy

▶ The gravitational potential energy $U(r)$ of a system of two particles, with masses M and m and separated by a distance r, is the negative of the work that would be done by the gravitational force of either particle acting on the other if the separation between the particles were changed from infinite (very large) to r. This energy is

$$U = -\frac{GMm}{r}.$$

▶ If a system contains more than two particles, its total gravitational potential energy U is the sum of the terms representing the potential energies of all the pairs. For n particles, of masses m_i,

$$U = -\sum_{1 \le i < j \le n} \frac{Gm_i m_j}{r_{ij}^2}.$$

▶ An object will escape the gravitational pull of an astronomical body of mass M and radius R (that is, it will reach an infinite distance) if the object's speed near the body's surface is at least equal to the escape speed, given by

$$v = \sqrt{\frac{2GM}{R}}.$$

5.6 Planets and Satellites: Kepler's Laws

★ **Theorem 5.6.1**

(Kepler's Laws) The motion of satellites, both natural and artificial, is governed by Kepler's laws:

1. **The law of orbits.** All planets move in elliptical orbits with the Sun at one focus.

2. **The law of areas.** A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)

3. **The law of periods.** The square of the period T of any planet is proportional to the cube of the semimajor axis a of its orbit. For circular orbits with radius r,

$$T^2 = \frac{4\pi r^3}{GM},$$

where M is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis a is substituted for r.

5.7 Satellites: Orbits and Energy

▶ When a planet or satellite with mass m moves in a circular orbit with radius r, its potential energy U and kinetic energy K are given by

$$U = -\frac{GMm}{r} \text{ and } K = \frac{GMm}{2r}.$$

The mechanical energy $E = K + U$ is then

$$E = -\frac{GMm}{2r}.$$

5.8 Einstein and Gravitation

Einstein pointed out that gravitation and acceleration are equivalent. This principle of equivalence led him to a theory of gravitation (the general theory of relativity) that explains gravitational effects in terms of a curvature of space.

Einstein Field Equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

where $R_{\mu\nu}$ is ricci tensor, $T_{\mu\nu}$ is energy momentum tensor and $g_{\mu\nu}$ is metric.

Chapter 6

Fluids

6.1 Fluids, Density, and Pressure

The density ρ of any material is defined as the material's mass per unit volume $\rho = \frac{\Delta m}{\Delta V}$. Usually, where a material sample is much larger than atomic dimensions, we can write this as $\rho = \frac{m}{V}$.

A fluid is a substance that can flow; it conforms to the boundaries of its container because it cannot withstand shearing stress. It can, however, exert a force perpendicular to its surface. That force is described in terms of pressure p: $p = \frac{\Delta F}{\Delta A}$ in which ΔF is the force acting on a surface element of area ΔA. If the force is uniform over a flat area, this can be written as $p = \frac{F}{A}$.

The force resulting from fluid pressure at a particular point in a fluid has the same magnitude in all directions.

6.2 Fluids at Rest

▶ Pressure in a fluid at rest varies with vertical position y. For y measured positive upward,

$$p_2 = p_1 + \rho g(y_2 - y_1).$$

If h is the depth of a fluid sample below some reference level at which the pressure is p_0 , this equation becomes

$$p = p_0 + \rho g h.$$

▶ Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure.

6.3 Measuring Pressure

▶ A mercury barometer can be used to measure atmospheric pressure.

▶ An open-tube manometer can be used to measure the gauge pressure of a confined gas.

6.4 Pascal's Principle

▶ Pascal's principle states that a change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

6.5 Archimedes' Principle

▶ Archimedes' principle states that when a body is fully or partially submerged in a fluid, the fluid pushes upward with a buoyant force with magnitude $F_b = m_f g$ where m_f is the mass of the fluid that has been pushed out of the way by the body.

▶ When a body floats in a fluid, the magnitude F_b of the (upward) buoyant force on the body is equal to the magnitude F_g of the (downward) gravitational force on the body.

▶ The apparent weight of a body on which a buoyant force acts is related to its actual weight by $weight_{app} = weight - F_b$.

6.6 The Equation of Continuity

▶ An ideal fluid is incompressible and lacks viscosity, and its flow is steady and irrotational.

▶ A streamline is the path followed by an individual fluid particle.

▶ A tube of flow is a bundle of streamlines.

▶ The flow within any tube of flow obeys the equation of continuity $R_V = Av = constant$, in which R_V is the volume flow rate, A is the cross-sectional area of the tube of flow at any point, and v is the speed of the fluid at that point.

▶ The mass flow rate R_m is $R_m = \rho R_V = \rho A v = constant$.

6.7 Bernoulli's Equation

★ Theorem 6.7.1

Applying the principle of conservation of mechanical energy to the flow of an ideal fluid leads to Bernoulli's equation:

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant$$

along any tube of flow.

Chapter 7

Oscillations

7.1 Simple Harmonic Motion

▶ The frequency f of periodic, or oscillatory, motion is the number of oscillations per second. In the SI system, it is measured in hertz: $1Hz = 1s^{-1}$.

▶ The period T is the time required for one complete oscillation, or cycle. It is related to the frequency by $T = \frac{1}{f}$.

▶ In simple harmonic motion (SHM), the displacement $x(t)$ of a particle from its equilibrium position is described by the equation $x = x_m \cos(\omega t + \phi)$, in which x_m is the amplitude of the displacement, $\omega t + \phi$ is the phase of the motion, and f is the phase constant. The angular frequency ω is related to the period and frequency of the motion by $\omega = 2\pi f = 2\pi/T$.

▶ Differentiating $x(t)$ leads to equations for the particle's SHM velocity and acceleration as functions of time: $v = -\omega x_m \sin(\omega t + \phi)$ and $a = -\omega^2 x_m \cos(\omega t + \phi)$. In the velocity function, the positive quantity ωx_m is the velocity amplitude v_m. In the acceleration function, the positive quantity $\omega^2 x_m$ is the acceleration amplitude a_m.

▶ A particle with mass m that moves under the influence of a Hooke's law restoring force given by $F = kx$ is a linear simple harmonic oscillator with $\omega = \sqrt{\dfrac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$.

Actually, we have ODE: $\dfrac{d^2x}{dt^2} = \ddot{x} = -\omega_0^2 x$.

7.2 Energy in Simple Harmonic Motion

▶ A particle in simple harmonic motion has, at any time, kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$ and potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$.

▶ If no friction is present, the mechanical energy $E = K + U = \frac{1}{2}kx_m^2$ remains constant even though K and U change.

7.3 An Angular Simple Harmonic Oscillator

▶ Torsion pendulum: Angular simple harmonic motion, we have $\tau = -\kappa\theta = I\dfrac{d^2\theta}{dt^2}$ with $\tau = I\alpha$. So we have $T = 2\pi\sqrt{\dfrac{I}{\kappa}}$.

7.4 Pendulums, circular motion

▶ A simple pendulum consists of a rod of negligible mass that pivots about its upper end, with a particle (the bob) attached at its lower end. If the rod swings through only small angles, its motion is approximately simple harmonic motion with a period given by $T = 2\pi\sqrt{\dfrac{I}{mgL}}$ since $\sin\theta \approx \theta$ when θ is small enough where I is the particle's rotational inertia about the pivot, m is the particle' s mass, and L is the rod's length.

▶ A physical pendulum has a more complicated distribution of mass. For small angles of swinging, its motion is simple harmonic motion with a period given by $T = 2\pi\sqrt{\dfrac{I}{mgh}}$ where I is the pendulum's rotational inertia about the pivot, m is the pendulum's mass, and h is the distance between the pivot and the pendulum's center of mass.

▶ Simple harmonic motion corresponds to the projection of uniform circular motion onto a diameter of the circle.

7.5 Damped Simple Harmonic Motion

▶ The mechanical energy E in a real oscillating system decreases during the oscillations because external forces, such as a drag force, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be damped.

▶ If the damping force is given by $\boldsymbol{F}_d = -b\boldsymbol{v}$, where $\boldsymbol{v}$is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by $m\ddot{x} + b\dot{x} + kx = 0$ then $x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$ where ω', the angular frequency of the damped oscillator, is given by $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$.

▶ If the damping constant is small ($b \ll \sqrt{km}$), then $\omega' = \omega$, where ω is the angular frequency of the undamped oscillator. For small b, the mechanical energy E of the oscillator is given by $E(t) = \frac{1}{2}kx_m^2 e^{-bt/m}$.

7.6 Forced Oscillations and Resonance

▶ If an external driving force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ω_d.

▶ The velocity amplitude v_m of the system is greatest when $\omega_d = \omega$, a condition called resonance. The amplitude x_m of the system is (approximately) greatest under the same condition.

Chapter 8

Waves I

8.1 Transverse Waves

▶ **Key Ideas 8.1.1**

Mechanical waves can exist only in material media and are governed by Newton's laws. Transverse mechanical waves, like those on a stretched string, are waves in which the particles of the medium oscillate perpendicular to the wave's direction of travel. Waves in which the particles of the medium oscillate parallel to the wave's direction of travel are longitudinal waves.

▶ **Key Ideas 8.1.2**

A sinusoidal wave moving in the positive direction of an x axis has the mathematical form

$$y(x,t) = y_m \sin(kx - \omega t),$$

where y_m is the amplitude (magnitude of the maximum displacement) of the wave, k is the angular wave number, ω is the angular frequency, and $kx - \omega t$ is the phase. The wavelength λ is related to k by $k = \frac{2\pi}{\lambda}$.

▶ **Key Ideas 8.1.3**

The period T and frequency f of the wave are related to ω by $\frac{\omega}{2\pi} = f = \frac{1}{T}$.

▶ **Key Ideas 8.1.4**

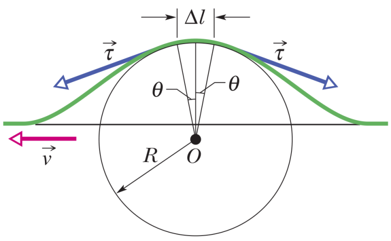
The wave speed v (the speed of the wave along the string) is related to these other parameters by $v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$.

▶ Key Ideas 8.1.5

Any function of the form $y(x,t) = h(kx \pm \omega t)$ can represent a traveling wave with a wave speed as given above and a wave shape given by the mathematical form of h. The plus sign denotes a wave traveling in the negative direction of the x axis, and the minus sign a wave traveling in the positive direction.

8.2 Wave Speed on a Stretched String

By Newton's law, we have $F = ma$, hence $2\tau \sin\theta = m\frac{v^2}{R}$. We let μ be linear density, and $\theta \approx \sin\theta$ since θ is small enough. Then $v = \sqrt{\frac{\tau}{\mu}}$.



▶ Key Ideas 8.2.1

The speed of a wave on a stretched string is set by properties of the string, not properties of the wave such as frequency or amplitude. The speed of a wave on a string with tension τ and linear density μ is

$$v = \sqrt{\frac{\tau}{\mu}}.$$

8.3 Energy and Power of a Wave Traveling Along a String

▶ Key Ideas 8.3.1

The average kinetic energy of a sinusoidal wave on a stretched string is given by $\frac{1}{4}\mu v\omega^2 y_m^2$.

Proof. The kinetic energy dK associated with a string element of mass dm is given by $dK = \frac{1}{2}dmu^2$ where u is the transverse speed of the oscillating string element. To find u, we have $u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$. Since $dm = \mu dx$, hence $\frac{dK}{dt} = \frac{1}{2}\mu v\omega^2 y_m^2 \cos(kx - \omega t)$, thus $\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{4}\mu v\omega^2 y_m^2$. □

▶ Key Ideas 8.3.2

The average elastic potential energy of a sinusoidal wave on a stretched string is given by $\frac{1}{4}\mu v\omega^2 y_m^2$.

Proof. Since $U = \frac{1}{2}k'y^2$ and $k' = m\omega^2$, we have $U = \frac{1}{2}m\omega^2 y^2$. Then $dU = \frac{1}{2}\mu\omega^2 y_m^2 \sin^2(kx - \omega t)dx$. Let $t = 0$, we have $U_\lambda = \int_0^\lambda \frac{1}{2}\mu\omega^2 y_m^2 \sin^2(kx)dx = \frac{1}{4}\mu\omega^2 y_m^2 \lambda$, thus $\frac{U_\lambda}{T} = \frac{1}{4}\mu v\omega^2 y_m^2$. □

Notation 8.3.1. Actually, the average elastic potential energy of a sinusoidal wave on a stretched string is equal to the average kinetic energy of a sinusoidal wave on a stretched string.

▶ **Key Ideas 8.3.3**

The average power, which is the average rate at which energy of both kinds is transmitted by the wave, is then

$$P_{avg} = 2\left(\frac{dK}{dt}\right)_{avg} = \frac{2U_\lambda}{T} = \frac{1}{2}\mu v\omega^2 y_m^2.$$

8.4 The Wave Equation

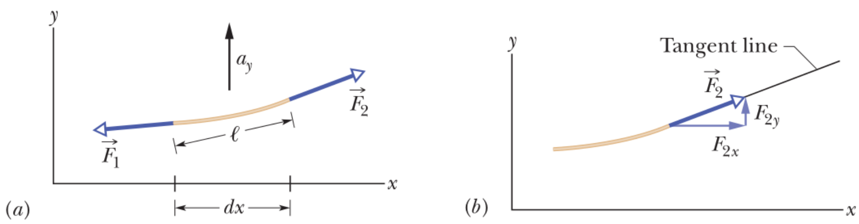
★ **Theorem 8.4.1**

The general differential equation that governs the travel of waves of all types is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}.$$

Here the waves travel along an x axis and oscillate parallel to the y axis, and they move with speed v, in either the positive x direction or the negative x direction.

Proof. Look at this picture:



By Newton's second law, we have $F_{2y} - F_{1y} = a_y dm$, and $dm = \mu dx, a_y = \frac{\partial^2 y}{\partial t^2}$. And $\frac{F_{2y}}{F_{2x}} = S_2$ and since $F_{2y} \ll F_{2x}$, thus $F_2 := \tau = \sqrt{F_{2x}^2 + F_{2y}^2} \approx F_{2x}$. Hence $F_{2y} = \tau S_2$ and $F_{1y} = \tau S_1$. So we have $\tau S_2 - \tau S_1 = (\mu dx)\frac{\partial^2 y}{\partial t^2}$, hence $\frac{\partial S}{\partial x} = \frac{\mu}{\tau}\frac{\partial^2 y}{\partial t^2}$ and $\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{\tau}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$. □

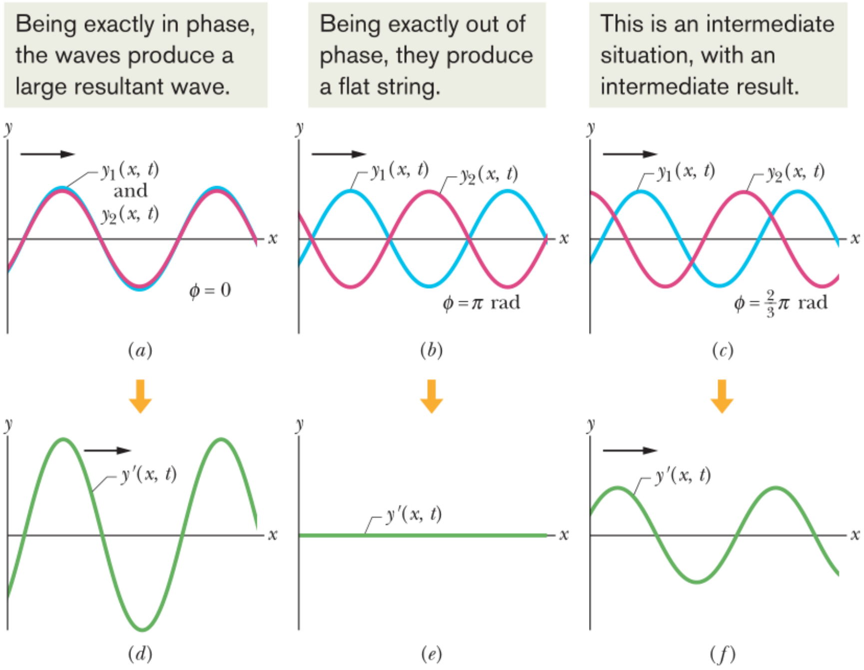
8.5 Interference of Waves

Let one wave traveling along a stretched string be given by $y_1(x,t) = y_m \sin(kx - \omega t)$, and another,shifted from the first, by $y_1(x,t) = y_m \sin(kx - \omega t + \phi)$. So $y'(x,t) = y_1(x,t) + y_2(x,t) = 2y_m \cos \frac{1}{2}\phi \sin(kx - \omega t + \frac{1}{2}\phi)$.

Some special situations:

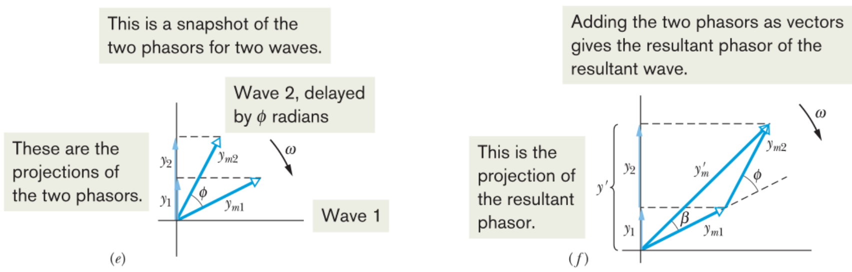
8.6 Phasors

A wave $y(x,t)$ can be represented with a phasor. This is a vector that has a magnitude equal to the amplitude y_m of the wave and that rotates about an origin with an angular speed equal to the angular



frequency ω of the wave. The projection of the rotating phasor on a vertical axis gives the displacement y of a point along the wave's travel.

Use this we can add like this:



8.7 Standing Waves and Resonance

If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

Let $y_1 = y_m \sin(kx - \omega t)$ and $y_2 = y_m \cos(kx - \omega t)$, then $y' = y_1 + y_2 = 2y_m \sin kx \cos \omega t$.

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a resonant frequency, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length L with fixed ends, the resonant frequencies are $f = \frac{v}{\lambda} = n\frac{v}{2L}$.

The oscillation mode corresponding to $n = 1$ is called the fundamental mode or the first harmonic; the mode corresponding to $n = 2$ is the second harmonic; and so on.

Chapter 9

Waves II

9.1 Speed of Sound

▶ **Key Ideas 9.1.1**

Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of a sound wave in a medium having bulk modulus $\boldsymbol{B}$ and density ρ is $v = \sqrt{\frac{\boldsymbol{B}}{\rho}}$, where $\boldsymbol{B} = -\frac{\Delta p}{\Delta V/V}$.

9.2 Traveling Sound Waves

▶ **Key Ideas 9.2.1**

A sound wave causes a longitudinal displacement s of a mass element in a medium as given by $s = s_m \cos(kx - \omega t)$ where s_m is the displacement amplitude (maximum displacement) from equilibrium, $k = \frac{2\pi}{\lambda}$, and $v = 2\pi f$, λ and f being the wavelength and frequency, respectively, of the sound wave.

▶ **Key Ideas 9.2.2**

The sound wave also causes a pressure change of the medium from the equilibrium pressure $\Delta p = \boldsymbol{B} k s_m \sin(kx - \omega t)$ where the pressure amplitude is $\Delta p_m = v\rho\omega s_m$.

Since $\Delta p = -\boldsymbol{B}\frac{\Delta V}{V}$ and $V = A\Delta x, \Delta V = A\Delta s$, so $\Delta p = -\boldsymbol{B}\frac{\partial s}{\partial x} = \boldsymbol{B} k s_m \sin(kx - \omega t)$. So we have $\Delta p_m = v\rho\omega s_m$ since $v = \sqrt{\frac{\boldsymbol{B}}{\rho}}$.

9.3 Interference

Let $s_1 = s_m \cos(kx - \omega t)$ and $s_2 = s_m \cos(kx - \omega t + \phi)$, thus $s' = 2s_m \cos \frac{1}{2}\phi s_m \cos(kx - \omega t + \frac{1}{2}\phi)$.

▶ Key Ideas 9.3.1

The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference ϕ there. If the sound waves were emitted in phase and are traveling in approximately the same direction, ϕ is given by $\phi = \frac{\Delta L}{\lambda} 2\pi$ where ΔL is their path length difference.

▶ Key Ideas 9.3.2

Fully constructive interference occurs when ϕ is an integer multiple of 2π, $\phi = 2n\pi, n = 0, 1, 2, ...$, and, equivalently, when ΔL is related to wavelength λ by $\frac{\Delta L}{\lambda} = 0, 1, 2, ...$

▶ Key Ideas 9.3.3

Fully destructive interference occurs when ϕ is an odd multiple of π, $\phi = (2n + 1)\pi, n = 0, 1, 2, ...$, and $\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, ...$

9.4 Intensity and Sound Level

▶ Key Ideas 9.4.1

The intensity I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface: $I = \frac{P}{A}$, where P is the time rate of energy transfer (power) of the sound wave and A is the area of the surface intercepting the sound. The intensity I is related to the displacement amplitude s_m of the sound wave by $I = \frac{1}{2}\rho v \omega^2 s_m^2$.

▶ Key Ideas 9.4.2

The intensity at a distance r from a point source that emits sound waves of power P_s equally in all directions (isotropically) is $I = \frac{P_s}{4\pi r^2}$.

▶ Key Ideas 9.4.3

The sound level β in decibels (dB) is defined as $\beta = (10dB) \log \frac{I}{I_0}$ where $I_0 (= 10^{-12} W/m^2)$ is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

9.5 Sources of Musical Sound

▶ Key Ideas 9.5.1

Standing sound wave patterns can be set up in pipes (that is, resonance can be set up) if sound of the proper wavelength is introduced in the pipe.

▶ Key Ideas 9.5.2

A pipe open at both ends will resonate at frequencies $f = \dfrac{v}{\lambda} = \dfrac{nv}{2L}, n = 1, 2, 3, ...$ where v is the speed of sound in the air in the pipe.

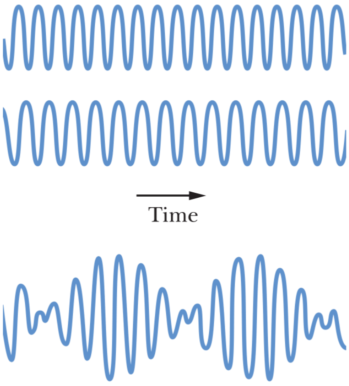
▶ Key Ideas 9.5.3

For a pipe closed at one end and open at the other, the resonant frequencies are $f = \dfrac{v}{\lambda} = \dfrac{nv}{4L}, n = 1, 3, 5, ...$

9.6 Beats

▶ Example 9.6.1

If we listen, a few minutes apart, to two sounds whose frequencies are, say, 552 and 564 Hz,most of us cannot tell one from the other because the frequencies are so close to each other. However, if the sounds reach our ears simultaneously, what we hear is a sound whose frequency is 558 Hz, the average of the two combining frequencies. We also hear a striking variation in the intensity of this sound—it increases and decreases in slow, wavering beats that repeat at a frequency of 12 Hz, the difference between the two combining frequencies.



Let $s_1 = s_m \cos\omega_1 t$ and $s_2 = s_m \cos\omega_2 t$, we have $s = s_1 + s_2 = 2s_m \cos\omega' t \cos\omega t$ where $\omega = \frac{1}{2}(\omega_1 + \omega_2)$ and $\omega' = \frac{1}{2}(\omega_1 - \omega_2)$. We have $\omega_{beat} = 2\omega' = \omega_1 - \omega_2$, hence $f_{beat} = f_1 - f_2$.

9.7 The Doppler Effect

▶ **Key Ideas 9.7.1**

The Doppler effect is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency f' is given in terms of the source frequency f by

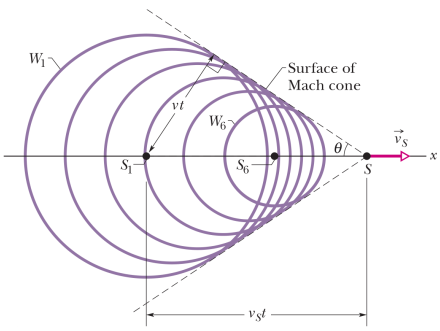
$$f' = f\frac{v \pm v_D}{v \mp v_S},$$

where v_D is the speed of the detector relative to the medium (If it is close to the emission source, the front operation symbol is $+$ sign, otherwise $-$ sign), v_S is that of the source (If it is close to the detector, the operation symbol in front is $-$ sign, otherwise it is $+$ sign), and v is the speed of sound in the medium.

9.8 Supersonic Speeds, Shock Waves

▶ **Key Ideas 9.8.1**

If the speed of a source relative to the medium exceeds the speed of sound in the medium, the Doppler equation no longer applies. In such a case, shock waves result. The half-angle θ of the Mach cone is given by $\sin\theta = \frac{v}{v_S}$.



Chapter 10

Temperature, Heat, and the First Law of Thermodynamics

10.1 Temperature

▶ Key Ideas 10.1.1

Temperature is an SI base quantity related to our sense of hot and cold. It is measured with a thermometer, which contains a working substance with a measurable property, such as length or pressure, that changes in a regular way as the substance becomes hotter or colder.

▶ Key Ideas 10.1.2

When a thermometer and some other object are placed in contact with each other, they eventually reach thermal equilibrium. The reading of the thermometer is then taken to be the temperature of the other object. The process provides consistent and useful temperature measurements because of the zeroth law of thermodynamics: If bodies A and B are each in thermal equilibrium with a third body C (the thermometer), then A and B are in thermal equilibrium with each other.

▶ Key Ideas 10.1.3

In the SI system, temperature is measured on the Kelvin scale, which is based on the triple point of water (273.16 K). Other temperatures are then defined by use of a constant-volume gas thermometer, in which a sample of gas is maintained at constant volume so its pressure is proportional to its temperature. We define the temperature T as measured with a gas thermometer to be

$$T = (273.16K) \lim_{gas \to 0} \frac{p}{p_3}.$$

Here T is in kelvins, and p_3 and p are the pressures of the gas at $273.16K$ and the measured temperature, respectively.

★ **Theorem 10.1.1**

(**The Zeroth law of Thermodynamics**) If bodies A and B are each in thermal equilibrium with a third body T, then A and B are in thermal equilibrium with each other.

(Two bodies are in thermal equilibrium with each other if they have the same temperature.)

10.2 The Celsius and Fahrenheit Scales

▶ **Key Ideas 10.2.1**

The Celsius temperature scale is defined by $T_C = T - 273.15°$, with T in kelvins. The Fahrenheit temperature scale is defined by $T_F = \frac{9}{5}T_C + 32°$.

Notation 10.2.1. Actually, the triple point of water is $273.16K$ and $0.01°C$ and $32.02°F$.

Notation 10.2.2. Temperature difference denoted by $C°$ or $F°$. But in China, this is banned.

10.3 Thermal Expansion

▶ **Key Ideas 10.3.1**

All objects change size with changes in temperature. For a temperature change ΔT, a change ΔL in any linear dimension L is given by

$$\Delta L = L\alpha\Delta T,$$

in which α is the coefficient of linear expansion.

▶ **Key Ideas 10.3.2**

The change ΔV in the volume V of a solid or liquid is

$$\Delta V = V\beta\Delta T.$$

Here $\beta = 3\alpha$ is the material's coefficient of volume expansion.

10.4 Absorption of Heat

▶ **Key Ideas 10.4.1**

Heat Q is energy that is transferred between a system and its environment because of a temperature difference between them. It can be measured in joules (J), calories (cal), kilocalories (Cal or kcal), or British thermal units (Btu), with

$$1cal = 3.968 \times 10^{-3} Btu = 4.1868 J.$$

▶ **Key Ideas 10.4.2**

If heat Q is absorbed by an object, the object's temperature change $T_f - T_i$ is related to Q by

$$Q = C(T_f - T_i),$$

in which C is the heat capacity of the object. If the object has mass m, then

$$Q = cm(T_f - T_i),$$

where c is the specific heat of the material making up the object.

▶ **Key Ideas 10.4.3**

The molar specific heat of a material is the heat capacity per mole, which means per 6.02×10^{23} elementary units of the material.

▶ **Key Ideas 10.4.4**

Heat absorbed by a material may change the material's physical state—for example, from solid to liquid or from liquid to gas. The amount of energy required per unit mass to change the state (but not the temperature) of a particular material is its heat of transformation L. Thus,

$$Q = Lm.$$

▶ **Key Ideas 10.4.5**

The heat of vaporization L_V is the amount of energy per unit mass that must be added to vaporize a liquid or that must be removed to condense a gas.

▶ **Key Ideas 10.4.6**

The heat of fusion L_F is the amount of energy per unit mass that must be added to melt a solid or that must be removed to freeze a liquid.

10.5 The First Law of Thermodynamics

▶ **Key Ideas 10.5.1**

A gas may exchange energy with its surroundings through work. The amount of work W done by a gas as it expands or contracts from an initial volume V_i to a final volume V_f is given by

$$W = \int_{\Delta W} dW = \int_{V_i}^{V_f} pdV.$$

The integration is necessary because the pressure p may vary during the volume change.

▶ **Key Ideas 10.5.2**

The principle of conservation of energy for a thermodynamic process is expressed in the first law of thermodynamics, which may assume either of the forms $\Delta E_{int} = E_{int,f} - E_{int,i} = Q - W$ or $dE_{int} = dQ - dW$. E_{int} represents the internal energy of the material, which depends only on the material's state (temperature, pressure, and volume). Q represents the energy exchanged as heat between the system and its surroundings; Q is positive if the system absorbs heat and negative if the system loses heat. W is the work done by the system; W is positive if the system expands against an external force from the surroundings and negative if the system contracts because of an external force.

Notation 10.5.1. Q and W are path dependent; ΔE_{int} is path independent.

▶ **Key Ideas 10.5.3**

The first law of thermodynamics finds application in several special cases:

(i)adiabatic processes: $Q = 0, \Delta E_{int} = -W$;

(ii)constant-volume processes: $W = 0, \Delta E_{int} = Q$;

(iii)cyclical processes: $\Delta E_{int} = 0, Q = W$;

(iv)free expansions: $Q = W = \Delta E_{int} = 0$.

10.6 Heat Transfer Mechanisms

▶ **Key Ideas 10.6.1**

The rate P_{cond} at which energy is conducted through a slab for which one face is maintained at the higher temperature T_H and the other face is maintained at the lower temperature T_C is

$$P_{cong} = \frac{Q}{t} = kA\frac{T_H - T_C}{L}.$$

Here each face of the slab has area A, the length of the slab (the distance between the faces) is L, and k is the thermal conductivity of the material.

▶ Key Ideas 10.6.2

The concept of thermal resistance R has been introduced into engineering practice.The R-value of a slab of thickness L is defined as

$$R = \frac{L}{k}.$$

▶ Key Ideas 10.6.3

Any number n of materials making up a slab:

$$P_{cong} = \frac{A(T_H - T_C)}{\sum(L/k)}.$$

▶ Key Ideas 10.6.4

Radiation is an energy transfer via the emission of electromagnetic energy. The rate P_{rad} at which an object emits energy via thermal radiation is

$$P_{rad} = \sigma\varepsilon AT^4,$$

where σ $(= 5.6704 \times 10^{-8} W/m^2 \cdot K^4)$ is the Stefan−Boltzmann constant, ε is the emissivity of the object's surface, A is its surface area, and T is its surface temperature (in kelvins). The rate P_{abs} at which an object absorbs energy via thermal radiation from its environment, which is at the uniform temperature T_{env} (in kelvins), is

$$P_{abs} = \sigma\varepsilon AT_{env}^4.$$

Chapter 11

The Kinetic Theory of Gases

11.1 Avogadro's Number

▶ **Key Ideas 11.1.1**

The kinetic theory of gases relates the macroscopic properties of gases (for example, pressure and temperature) to the microscopic properties of gas molecules (for example, speed and kinetic energy).

▶ **Key Ideas 11.1.2**

One mole of a substance contains N_A (Avogadro's number) elementary units (usually atoms or molecules), where N_A is found experimentally to be

$$N_A = 6.02 \times 10^{23} mol^{-1}.$$

One molar mass M of any substance is the mass of one mole of the substance.

▶ **Key Ideas 11.1.3**

A mole is related to the mass m of the individual molecules of the substance by

$$M = mN_A.$$

▶ **Key Ideas 11.1.4**

The number of moles n contained in a sample of mass M_{sam}, consisting of N molecules, is related to the molar mass M of the molecules and to Avogadro's number N_A as given by

$$n = \frac{N}{N_A} = \frac{M_{sam}}{M} = \frac{M_{sam}}{mN_A}.$$

11.2 Ideal Gases

▶ Key Ideas 11.2.1

An ideal gas is one for which the pressure p, volume V, and temperature T are related by

$$PV = nRT.$$

Here n is the number of moles of the gas present and R is a constant $(8.31 J/mol \cdot K)$ called the gas constant.

▶ Key Ideas 11.2.2

The ideal gas law can also be written as $PV = \kappa NT$ where the Boltzmann constant κ is

$$\kappa = \frac{R}{N_A} = 1.38 \times 10^{-23} J/K.$$

▶ Key Ideas 11.2.3

The work done by an ideal gas during an isothermal (constant-temperature) change from volume V_i to volume V_f is

$$W = nRT \ln \frac{V_f}{V_i}.$$

11.3 Pressure, Temperature, and RMS Speed

▶ Key Ideas 11.3.1

In terms of the speed of the gas molecules, the pressure exerted by n moles of an ideal gas is

$$p = \frac{nMv_{rms}^2}{3V}$$

where $v_{rms} = \sqrt{\sum(v_i^2/n)}$ is the root-mean-square speed of the molecules, M is the molar mass, and V is the volume.

Proof. At x component, we have $\Delta\mathfrak{p} = -2mv_x$ and then $\frac{\Delta\mathfrak{p}}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$. And $\boldsymbol{F} = \frac{d\mathfrak{p}}{dt}$ and

$$p_x = \frac{F}{L^2} = \frac{nMv_x^2}{V} \Rightarrow p = \frac{nMv^2}{3V} \text{ since } v^2 = \sum_{x,y,z} v_i^2.$$

And well done! □

▶ Key Ideas 11.3.2

The rms speed can be written in terms of the temperature as

$$v_{rms} = \sqrt{\frac{3RT}{M}}.$$

11.4 Translational Kinetic Energy

▶ **Key Ideas 11.4.1**

The average translational kinetic energy per molecule in an ideal gas is

$$K_{avg} = \frac{1}{2}mv_{rms}^2.$$

▶ **Key Ideas 11.4.2**

The average translational kinetic energy is related to the temperature of the gas:

$$K_{avg} = \frac{3}{2}\kappa T.$$

11.5 Mean Free Path

▶ **Key Ideas 11.5.1**

The mean free path λ of a gas molecule is its average path length between collisions and is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (N/V)} = \frac{\kappa T}{\sqrt{2}\pi d^2 p},$$

where N/V is the number of molecules per unit volume, d is the molecular diameter and p is pressure.

11.6 The Distribution of Molecular Speeds

▶ **Key Ideas 11.6.1**

The Maxwell speed distribution $P(v)$ is a function such that $P(v)dv$ gives the fraction of molecules with speeds in the interval dv at speed v:

$$P(v) = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

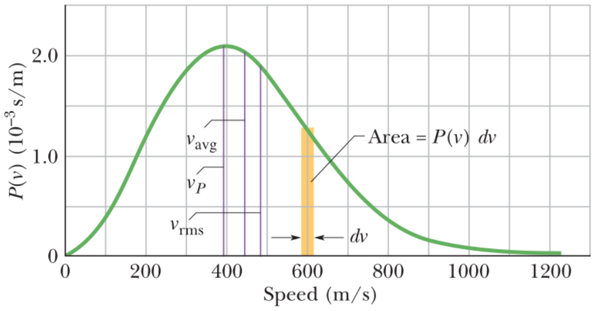
▶ **Key Ideas 11.6.2**

Three measures of the distribution of speeds among the molecules of a gas are

$$v_{avg} = \int_0^\infty vP(v)dv = \sqrt{\frac{8RT}{\pi M}}, (v^2)_{avg} = \int_0^\infty v^2 P(v)dv = \frac{3RT}{M},$$

and

$$\frac{dP}{dv} = 0 \Rightarrow v_P = \frac{2RT}{M}.$$



11.7 The Molar Specific Heats of an Ideal Gas

▶ **Key Ideas 11.7.1**

The molar specific heat C_V of a gas at constant volume is defined as

$$C_V = \frac{Q}{n\Delta T} = \frac{\Delta E_{in}}{n\Delta T},$$

in which Q is the energy transferred as heat to or from a sample of n moles of the gas, ΔT is the resulting temperature change of the gas, and ΔE_{int} is the resulting change in the internal energy of the gas.

▶ **Key Ideas 11.7.2**

For an ideal monatomic gas, $C_V = \frac{3}{2}R = 12.5J/mol \cdot K$.

▶ **Key Ideas 11.7.3**

The molar specific heat C_p of a gas at constant pressure is defined to be

$$C_p = \frac{Q}{n\Delta T},$$

in which Q, n, and ΔT are defined as above. C_p is also given by $C_p = C_V + R$.

▶ **Key Ideas 11.7.4**

For n moles of an ideal gas,

$$E_{int} = nC_V T.$$

▶ **Key Ideas 11.7.5**

If n moles of a confined ideal gas undergo a temperature change ΔT due to any process, the change in the internal energy of the gas is

$$\Delta E_{int} = nC_V \Delta T.$$

11.8 Degrees of Freedom and Molar Specific Heats

▶ **Key Ideas 11.8.1**

We find C_V by using the equipartition of energy theorem, which states that every degree of freedom of a molecule (that is, every independent way it can store energy) has associated with it—on average —an energy $\frac{1}{2}\kappa T$ per molecule ($\frac{1}{2}RT$ per mole).

▶ **Key Ideas 11.8.2**

If f is the number of degrees of freedom, then

$$E_{int} = \frac{f}{2}nRT, C_V = \frac{f}{2}R = 4.16f J/mol \cdot K.$$

▶ **Key Ideas 11.8.3**

For monatomic gases $f = 3$ (three translational degrees); for diatomic gases $f = 5$ (three translational and two rotational degrees).

11.9 The Adiabatic Expansion of an Ideal Gas

▶ **Key Ideas 11.9.1**

When an ideal gas undergoes a slow adiabatic volume change (a change for which $Q = 0$),

$$PV^\gamma = constant,$$

in which $\gamma(= C_p/C_V)$ is the ratio of molar specific heats for the gas.

(For a free expansion, $PV = constant$ or $T = constant$.)

Notation 11.9.1. With $pV = nRT$, we have $TV^{\gamma-1} = constant$.

Chapter 12

Entropy and the Second Law of Thermodynamics (No problems)

12.1 Entropy

▶ **Key Ideas 12.1.1**

An irreversible process is one that cannot be reversed by means of small changes in the environment. The direction in which an irreversible process proceeds is set by the change in entropy ΔS of the system undergoing the process. Entropy S is a state property (or state function) of the system; that is, it depends only on the state of the system and not on the way in which the system reached that state. The entropy postulate states (in part): If an irreversible process occurs in a closed system, the entropy of the system always increases.

▶ **Key Ideas 12.1.2**

The entropy change ΔS for an irreversible process that takes a system from an initial state i to a final state f is exactly equal to the entropy change ΔS for any reversible process that takes the system between those same two states. We can compute the latter (but not the former) with

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}.$$

Here Q is the energy transferred as heat to or from the system during the process, and T is the temperature of the system in kelvins during the process.

▶ **Key Ideas 12.1.3**

For a reversible isothermal process, the expression for an entropy change reduces to

$$\Delta S = S_f - S_i = \frac{Q}{T}.$$

▶ **Key Ideas 12.1.4**

When the temperature change ΔT of a system is small relative to the temperature (in kelvins) before and after the process, the entropy change can be approximated as

$$\Delta S = S_f - S_i \approx \frac{Q}{T_{avg}}.$$

▶ **Key Ideas 12.1.5**

When an ideal gas changes reversibly from an initial state with temperature T_i and volume V_i to a final state with temperature T_f and volume V_f, the change ΔS in the entropy of the gas is

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}.$$

▶ **Key Ideas 12.1.6**

The second law of thermodynamics, which is an extension of the entropy postulate, states: If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases. In equation form,

$$\Delta S \geq 0.$$

12.2 Entropy in the Real World: Engines

▶ **Key Ideas 12.2.1**

An engine is a device that, operating in a cycle, extracts energy as heat $|Q_H|$ from a high-temperature reservoir and does a certain amount of work $|W|$. The efficiency ε of any engine is defined as

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}.$$

▶ **Key Ideas 12.2.2**

In an ideal engine, all processes are reversible and no wasteful energy transfers occur due to, say, friction and turbulence.

▶ **Key Ideas 12.2.3**

A Carnot engine is an ideal engine that follows the cycle of Carnot. Its efficiency is

$$\varepsilon_C = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H},$$

in which T_H and T_L are the temperatures of the high- and low-temperature reservoirs, respectively. Real engines always have an efficiency lower than that of a Carnot engine. Ideal engines that are not Carnot engines also have efficiencies lower than that of a Carnot engine.

▶ **Key Ideas 12.2.4**

A perfect engine is an imaginary engine in which energy extracted as heat from the high-temperature reservoir is converted completely to work. Such an engine would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the absorption of energy as heat from a thermal reservoir and the complete conversion of this energy to work.

12.3 Refrigerators and Real Engines

▶ **Key Ideas 12.3.1**

A refrigerator is a device that, operating in a cycle, has work W done on it as it extracts energy Q_L as heat from a low-temperature reservoir. The coefficient of performance K of a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}.$$

▶ **Key Ideas 12.3.2**

A Carnot refrigerator is a Carnot engine operating in reverse. Its coefficient of performance is

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{T_L}{T_H - T_L}.$$

▶ **Key Ideas 12.3.3**

A perfect refrigerator is an entirely imaginary refrigerator in which energy extracted as heat from the low-temperature reservoir is somehow converted completely to heat discharged to the high-temperature reservoir without any need for work.

▶ **Key Ideas 12.3.4**

A perfect refrigerator would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the transfer of energy as heat from a reservoir at a given temperature to a reservoir at a higher temperature (without work being involved).

12.4 A Statistical View of Entropy

▶ **Key Ideas 12.4.1**

The entropy of a system can be defined in terms of the possible distributions of its molecules. For identical molecules, each possible distribution of molecules is called a microstate of the system. All equivalent microstates are grouped into a configuration of the system. The number of microstates in a configuration is the multiplicity W of the configuration.

▶ **Key Ideas 12.4.2**

For a system of N molecules that may be distributed between the two halves of a box, the multiplicity is given by

$$W = \frac{N!}{n_1!n_2!},$$

in which n_1 is the number of molecules in one half of the box and n_2 is the number in the other half. A basic assumption of statistical mechanics is that all the microstates are equally probable. Thus, configurations with a large multiplicity occur most often. When N is very large (say, $N = 10^{22}$molecules or more), the molecules are nearly always in the configuration in which $n_1 = n_2$.

▶ **Key Ideas 12.4.3**

The multiplicity W of a configuration of a system and the entropy S of the system in that configuration are related by Boltzmann's entropy equation:

$$S = \kappa \ln W,$$

where $\kappa = 1.38 \times 10^{-23} J/K$ is the Boltzmann constant.

▶ **Key Ideas 12.4.4**

More strong Stirling's approximation, let $\theta_n \in (0,1)$ and we have

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\frac{\theta_n}{12n}}.$$

▶ Key Ideas 12.4.5

When N is very large (the usual case), we can approximate $\ln N!$ with Stirling's approximation:

$$\ln N! \approx N \ln N - N.$$