Homework 4 – A Report on Runge-Kutta's method

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8th.Dec.2021

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Abstract

In this report, we use Runge-Kutta's method and Adams Fourth-Order Predictor-Corrector to approximate solution of $y' = 1 + (y - t)^2$ on $[2, 3]$.

Contents

1 Introductions

In this report, we use Runge-Kutta's method and Adams Fourth-Order Predictor-Corrector to approximate solution of $y' = 1 + (y-t)^2$ on $[2, 3]$.

2 Algorithm Description & Mathematical Derivation

2.1 Runge-Kutta's method

Theorem 2.1.1. *Suppose that $f(t, y)$ and all its partial derivatives of order less than or equal to $n+1$ are continuous on $D = \{(t, y) : a \leq t \leq b, c \leq y \leq d\}$ and let $(t_0, y_0) \in D$. For every $(t, y) \in D$, there is ξ between t and t_0 and μ between y, y_0 width*

$$f(t, y) = P_n(t, y) + R_n(t, y),$$

where

$$\begin{aligned} P_n(t, y) = & f(t_0, y_0) + \left[(t - t_0)\frac{\partial f}{\partial t}(t_0, y_0) + (y - y_0)\frac{\partial f}{\partial y}(t_0, y_0)\right] \\ & + \cdots \\ & + \left[\frac{1}{n!}\sum_{j=0}^{n}\binom{n}{j}(t - t_0)^{n-j}(y - y_0)^j \frac{\partial^n f}{\partial t^{n-j}\partial y^j}(t_0, y_0)\right] \end{aligned}$$

and

$$R_n(t, y) = \frac{1}{(n+1)!}\sum_{j=0}^{n+1}\binom{n+1}{j}(t - t_0)^{n+1-j}(y - y_0)^j \frac{\partial^{n+1} f}{\partial t^{n+1-j}\partial y^j}(\xi, \mu).$$

2.2 Adams Fourth-Order Predictor-Corrector

Adams Fourth-Order Predictor-Corrector is based on the fourth-order Adams-Bashforth method as predictor and on iteration of the Adam-Moulton method as corrector, with the starting values obtained from the fourth-order Runge-Kutta method.

Definition 2.2.1. *An m-step multistep method of solving the initial-value Problem*

$$y' = f(t, y), a \leq t \leq b, y(a) = \alpha,$$

has a difference equation for finding the approximation w_{i+1} at the mesh point t_{i+1} represented by the following equation, where m is an integer greater than 1:

$$\begin{aligned} w_{i+1} = & a_{m-1}w_i + a_{m-2}w_{i-1} + \cdots + a_0 w_{i+1-m} \\ & + h[b_m f(t_{i+1}, w_{i+1}) + b_{m-1}f(t_i, w_i) + \cdots + b_0 f(t_{i+1-m}, w_{i+1-m})], \end{aligned}$$

for $i = m-1, m, \cdots, N-1$, where $h = (b-a)/N$, the $a_0, ..., a_{m-1}$ and $b_0, ..., b_m$ are constants, and the starting values

$$w_0 = \alpha, w_1 = \alpha_2, ..., w_{m-1} = \alpha_{m-1}$$

are specified.

Remark 2.2.2. *When $b_m = 0$, this is called Adams-Bashforth technique and $b_m \neq 0$, this is called Adams-Moulton technique.*

3 Codes and Numerical Experiments

3.1 Runge-Kutta's method

Problem 3.1.1. *Use Runge-Kutta's method to approximate solution of* $y' = 1 + (y - t)^2$ *on* $[2, 3]$.

Solution. Consider the following code:

```
>> format long
>> t=2;w=1;h=0.1;
for i=1:10
   K1= h*(1+(t-w)^2);
   K2=h*(1+(t+h/2-w-K1/2)^2);
   K3=h*(1+(t+h/2-w-K2/2)^2);
   K4=h*(1+(t+h/2-w-K3)^2);
   w=w+(K1+2*K2+2*K3+K4)/6;
   a(1,i)=w;t=2+i*h;x(1,i)=t;
end
```

And the result as follows

x	**values**
2.10	1.189435778045385
2.20	1.364079699749434
2.30	1.527321535976260
2.40	1.681588810493533
2.50	1.828665612358321
2.60	1.969892531194408
2.70	2.106296403335376
2.80	2.238677027007300
2.90	2.367666624378014
3.00	2.493771545819090

Well done. □

3.2 Adams Fourth-Order Predictor-Corrector

Problem 3.2.1. *Use Adams Fourth-Order Predictor-Corrector to approximate solution of* $y' = 1 + (y-t)^2$ *on* $[2,3]$.

Solution. Consider the following codes.

```
>> format long
for i=4:10
   t=2+i*h;
   w=w3+h*(55*(1+(t3-w3)^2)-59*(1+(t2-w2)^2)+37*(1+(t1-w1)^2)-9*(1+(
       t0-w0)^2))/24;
   w=w3+h*(9*(1+(t-w)^2)+19*(1+(t3-w3)^2)-5*(1+(t2-w2)^2)+(1+(t1-w1)
       ^2))/24;
   t0=t1;t1=t2;t2=t3;w0=w1;w1=w2;w2=w3;t3=t;w3=w;
   b(1,i)=w;
end
```

And the result as follows

x	**values**
2.10	1.18943577804538
2.20	1.36407969974943
2.30	1.52732153597626
2.40	1.68507125058763
2.50	1.83181032905356
2.60	1.97367622113906
2.70	2.11060372348383
2.80	2.24340892560078
2.90	2.37275672322023
3.00	2.49916425399177

Well done. □

References

[1] Richard L. Burden, J. Douglas Faires, Annette M. Burden, *Numerical Analysis (Tenth Edition)*, Cengage Learning, 2014.